

Intrabeam collisions, gas and electron effects in intense beams

Outline:

1. Beam/beam coulomb collisions
2. Beam/gas scattering
3. Charge changing processes
4. Gas pressure instability
5. Electron cloud processes
6. Electron-ion instability

Gas and electron effects

-Effects are quite different depending on
 q, m of species being accelerated

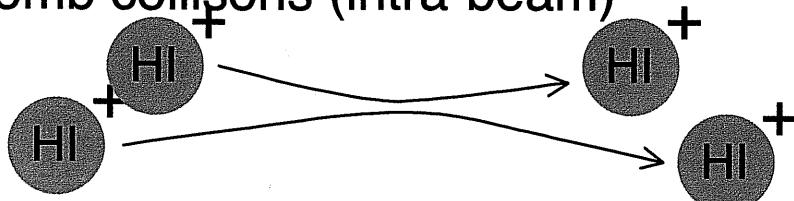
-Circular accelerators vs. Linacs
($t_{\text{residence}} \sim \text{ms to days}$ vs. 10's of μs)

-Long pulse vs. short pulse
($t_{\text{pulse}} \sim \text{10's of } \mu\text{s}$ vs. 10's of ns)

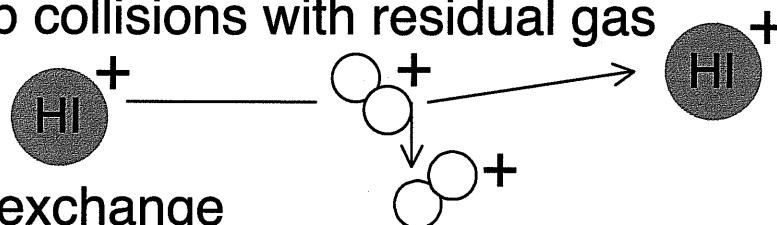
	Heavy ion		Residual gas molecule		e ⁻ electron
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Processes:

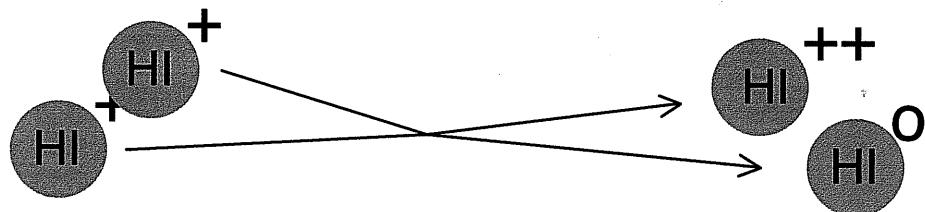
1. Coulomb collisions (intra-beam)



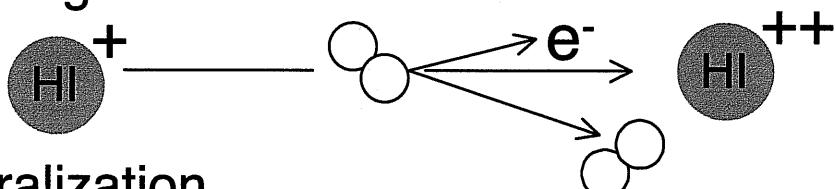
2. Coulomb collisions with residual gas



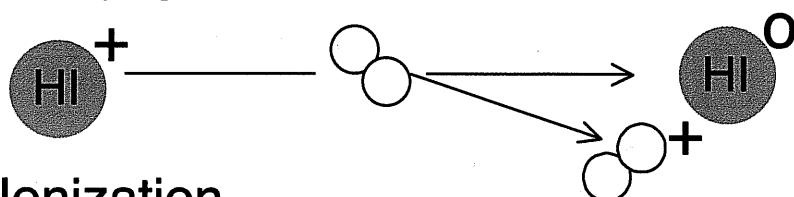
3. Charge exchange



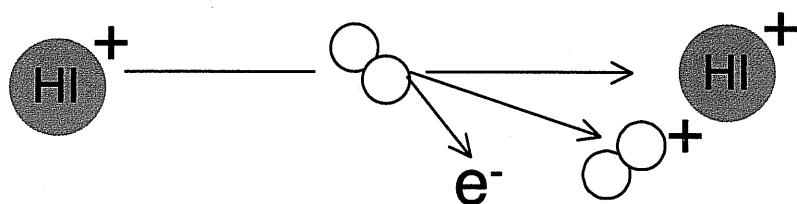
4. Stripping



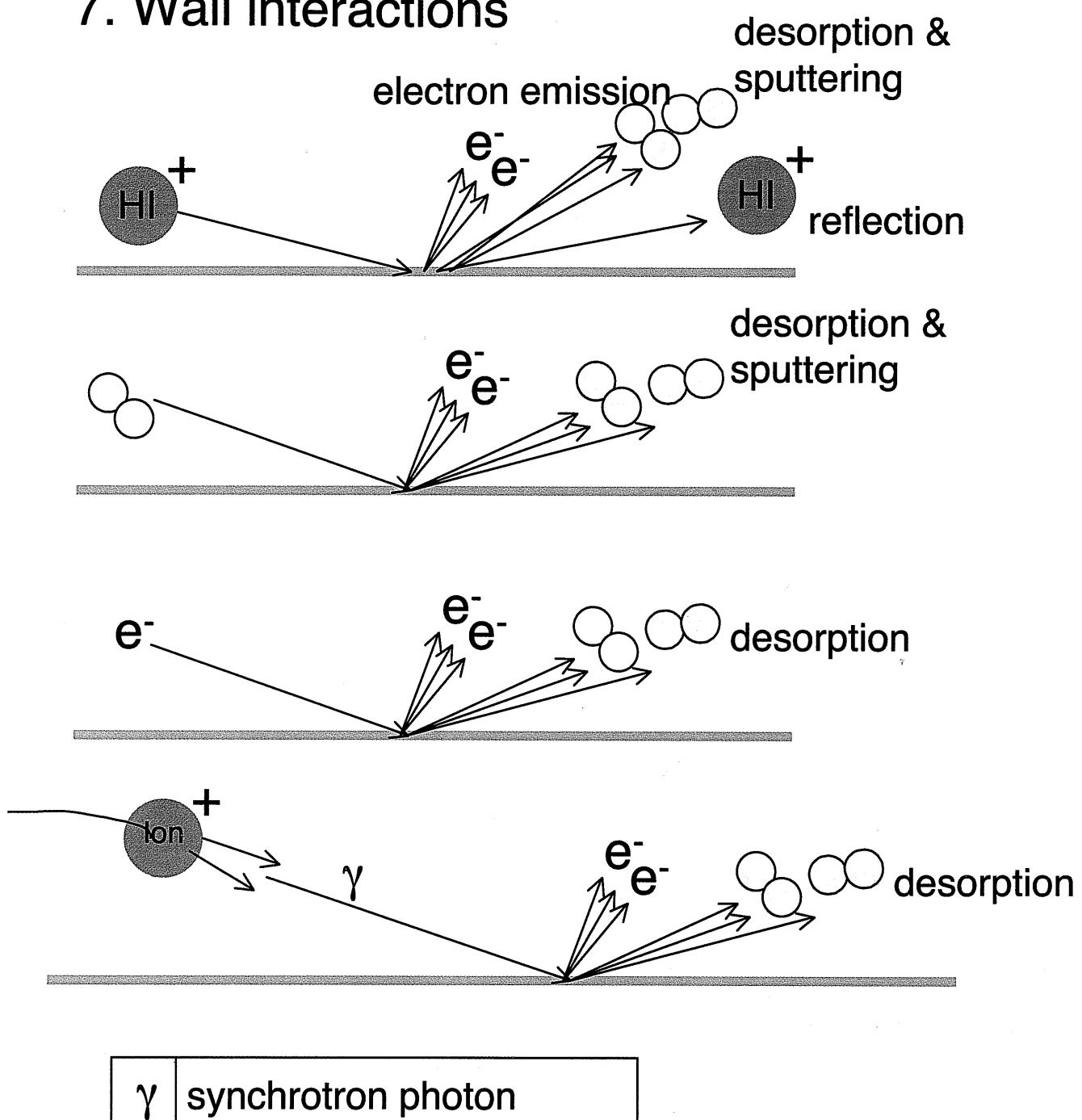
5. Neutralization



6. Gas Ionization



7. Wall interactions



I. COLLISIONS WITHIN BEAM REISON 6.4

CONSIDER EFFECTS OF COULOMB COLLISIONS

IN A CONTINUOUS BEAM PROPAGATING THROUGH

A SMOOTH FOCUSING CHANNEL WITH $T_{\perp 0} \neq T_{\parallel 0}$

(IF $T_{\perp 0} = T_{\parallel 0} \Rightarrow$ BEAM ALREADY RELAXED)

FROM ICHIMAKU & KOENBLUTH, PHYS FLUIDS 13, 2778, (1970) :

$$\frac{dT_{\perp}}{dt} = -\frac{1}{2} \frac{dT_{\parallel}}{dt} = -\frac{(T_{\perp} - T_{\parallel})}{\tau}$$

(since $T_x = T_y = T_{\perp}$, T_{\parallel} CHANGES AT TWICE THE RATE OF T_{\perp})

τ = RELAXATION TIME

$$= \frac{15 (k_b T_{\text{eff}} / m^2)^{3/2} (4\pi \epsilon_0)^2 m^2 c^3}{8\pi^{1/2} q^4 \ln \Lambda n} = \left(\frac{15 \pi^{1/2}}{8 k_b \tau} \right) v_c^{-1}$$

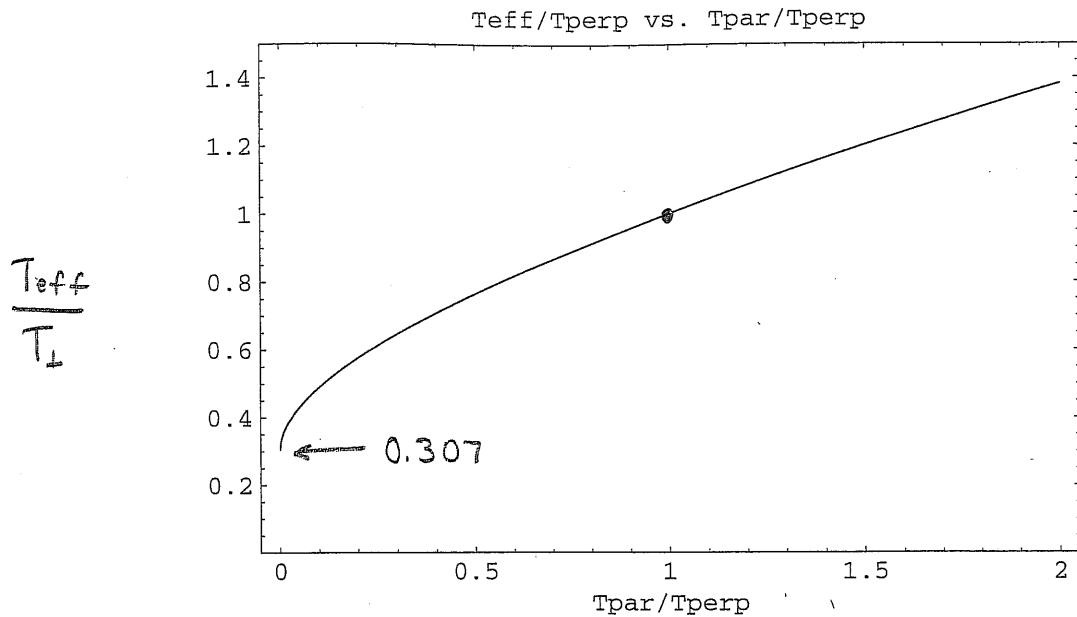
$$\ln \Lambda = \begin{cases} \ln \left(\frac{\epsilon_0 k T}{q^3 n^{1/2}} \right)^{3/2} & \text{for } \lambda_0 < a \\ \ln \frac{12\pi \epsilon_0 k T a}{q^2} & \text{for } \lambda_0 > a \end{cases}$$

COULOMB COLLISION APPROXIMATION

RATE FOR LARGE ANGLES
(PAGE 9 OF INTRODUCTION NOTES)

$$T_{\text{eff}} = T_{\perp} \left[\frac{15}{4} \int_{-1}^1 \frac{\mu^2 (1 - \mu^2) d\mu}{[(1 - \mu^2) + \mu^2 (T_{\parallel 0} / T_{\perp 0})]^{3/2}} \right]^{-2/3}$$

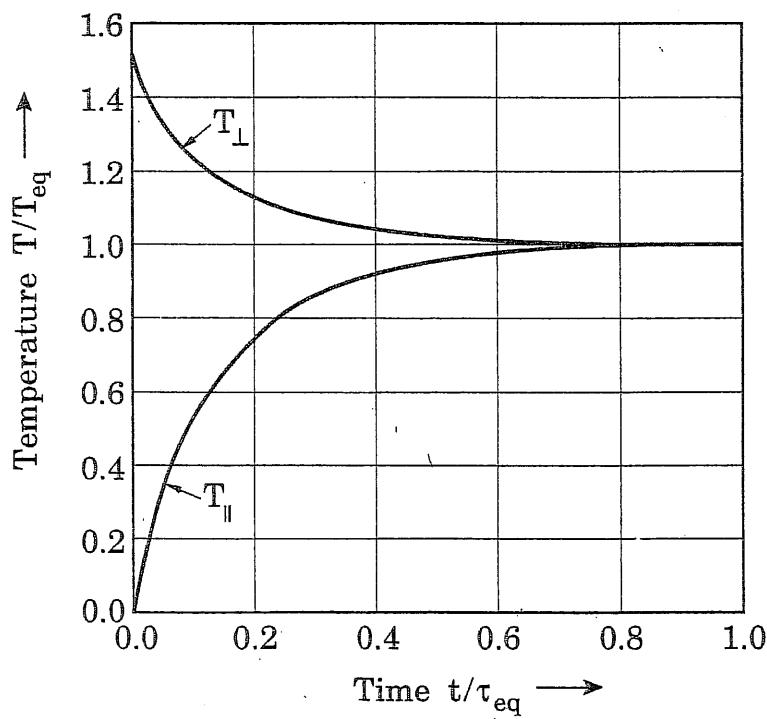
T_{eff} is an appropriate average of T_{\perp} & T_{\parallel}



$$T_{\perp} = \frac{2}{3} T_{\perp 0} \left(1 + \frac{1}{2} e^{-3t/\tau_{\text{eff}}} \right), \quad (6.156a)$$

$$T_{\parallel} = \frac{2}{3} T_{\perp 0} (1 - e^{-3t/\tau_{\text{eff}}}), \quad (6.156b)$$

(APPROXIMATE SOLUTIONS)



From
REISER p. 527

$$\tau_{\text{eq}} = \uparrow(T_{\text{eq}})$$

(7)

BOERSCH EFFECT

ARE COLLISIONS NEGLECTABLE? (NOT ALWAYS)

PUTTING IN NUMBERS:

FOR IONS:

$$\tau_{\text{eff}} = 4.3 \cdot 10^{-4} s \frac{(A^{1/2})}{Z^4} \left(\frac{kT_{\text{eff}}}{1 \text{ eV}} \right)^{3/2} \left(\frac{15}{n} \right) \left(\frac{10^{10} \text{ cm}^{-3}}{n} \right)$$

$$\ln \lambda = \ln \left[\frac{1.5 \cdot 10^5 (kT / 1 \text{ eV})^{3/2}}{Z^3 (n / 10^{10} \text{ cm}^{-3})} \right]$$

EXAMPLE: 2 MeV INJECTOR

$$\tau_{\text{eff}} \approx 8.8 \cdot 10^{-4} \text{ s} \quad \text{for } A = 39 \quad kT_{\text{eff}} = 0.3 \text{ eV}$$

$$Z = 1 \quad \ln \lambda = 8.5$$

$$n = 10^{10} \text{ cm}^{-3}$$

$$t_{\text{transit}} \approx \frac{2d}{v} \approx \frac{2(2m)}{(0.1) 3.10^8} = 1.3 \mu\text{s}$$

So $\tau_{\text{eff}} >> t_{\text{transit}} \Rightarrow$ collisions are rare BUT

$$T_{\text{accel}} = \frac{1}{2} \left(\frac{kT_0}{qV} \right) \quad kT_0 = 2.5 \cdot 10^{-9} \text{ eV} \quad \text{for } \begin{aligned} kT_0 &= 0.1 \text{ eV} \\ qV &= 2 \text{ MeV} \end{aligned}$$

$$T_{\text{collisions}} \approx \frac{2}{3} T_{J0} \left(1 - \exp(-3t/\tau_{\text{eff}}) \right) \approx 2T_{J0} \left(\frac{t_{\text{transit}}}{\tau_{\text{eff}}} \right) = .006 \text{ eV} \quad \text{for } T_{J0} = 1 \text{ eV}$$

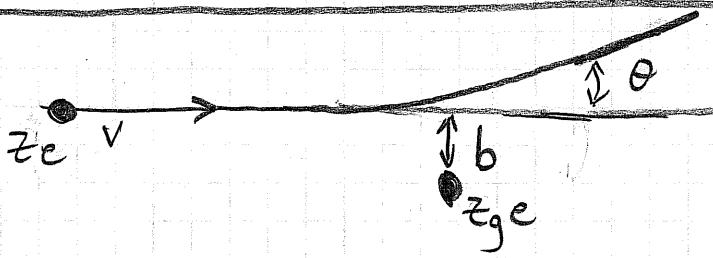
So T_h FROM "BOERSCH EFFECT"

>> T_h FROM LONGITUDINAL COOLING

(EVIDENTLY BOERSCH HAD FOUND ENERGY SPREAD
DOWNSTREAM FROM A WAIST DEPENDENT ON
BEAM CURRENT DENSITY AT WAIST, BUT
DID NOT ATTRIBUTE EFFECT TO COULOMB
COLLISIONS).

COULOMB COLLISIONS IN RESIDUAL GAS (REISER 6.4.3)

JACKSON CHAPTER 13



(RUTHERFORD)
SCATTERING)

$$\frac{dp}{dt} = \frac{Z Z_g e^2}{4\pi\epsilon_0 r^2} \frac{b}{r}$$

$$\Rightarrow \Delta p = \int_{-\infty}^{\infty} \frac{dp_x}{dt} dt \frac{dt}{dz} dz$$

$$= \frac{Z Z_g e^2 b}{4\pi\epsilon_0 v} \underbrace{\int_{-\infty}^{\infty} \frac{dz}{(z^2 + b^2)^{3/2}}}_{z/b} = \frac{2 Z Z_g e^2}{4\pi\epsilon_0 v b}$$

$$\frac{\Delta p}{P} \theta \approx \frac{\Delta p}{P} = \frac{2 Z Z_g e^2}{4\pi\epsilon_0 p v b} \Rightarrow \frac{db}{d\Omega} \sim \frac{1}{\theta^2}$$

DIFFERENTIAL CROSS SECTION FOR SCATTERING WITH IMPACT

PARAMETER b INTO SOLID ANGLE $d\Omega$ AT ANGLE θ SATISFIES

$$2\pi b db = \underbrace{\frac{d\Omega}{d\Omega} 2\pi \sin \theta d\theta}_{\text{AREA}} \underbrace{2\pi \sin \theta d\theta}_{\text{SOLID ANGLE}}$$

$$\Rightarrow \frac{d\Omega}{d\Omega} = \frac{b}{\sin \theta} \quad \left| \frac{db}{d\Omega} \right| = \left(\frac{2 Z Z_g e^2}{4\pi\epsilon_0 p v} \right)^2 \frac{1}{\theta^4}$$

ELECTRON
SCREENING
PUTS CUTOFF
AT SMALL θ
(LARGE b)

SO BETTER
TO USE

$$\frac{d\Omega}{d\Omega} = \left(\frac{2 Z Z_g e^2}{4\pi\epsilon_0 p v} \right)^2 \frac{1}{(\theta^2 + \theta_{\min}^2)^2}$$

AVERAGE ANGLE SQUARED FOR A SINGLE SCATTERING IS:

$$\bar{\theta}^2 = \frac{\int \theta^2 \frac{d\sigma}{d\Omega} 2\pi \sin \theta d\theta}{\int \frac{d\sigma}{d\Omega} 2\pi \sin \theta d\theta} \approx \frac{\int_{\theta_{\min}}^{\theta_{\max}} \frac{\theta^3}{(\theta^2 + \theta_{\min}^2)^2} d\theta}{\int_{\theta_{\min}}^{\theta_{\max}} \frac{\theta}{(\theta^2 + \theta_{\min}^2)^2} d\theta}$$

$$\approx 2\theta_{\min} \ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right)$$

↑
(I GOT 4, BUT
JACKSON GOT 2)

ASSUMES $\theta_{\max} \gg \theta_{\min}$
+ $\ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right) \gg 1$

MULTIPLE COLLISIONS

AFTER TRAVERSING DISTANCE s ,
AND UNDETECTING N_s COLLISIONS, THE
MEAN SQUARE ANGLE $\bar{\theta}^2$

$$\overline{\bar{\theta}^2} = N_s \bar{\theta}^2 = n_g \theta_s s \bar{\theta}^2$$

$$= 16 \pi n_g \left(\frac{Z Z_g e^2}{4 \pi \epsilon_0 m c^2 \gamma^2} \right)^2 \ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right) s$$

$$\left[\theta_s = \pi \left(\frac{Z Z_g e^2}{4 \pi \epsilon_0 m c^2 \gamma^2} \right)^2 \frac{1}{\theta_{\min}^2} \right]$$

JACKSON ARGUES θ_{\max} ARISES FROM DISTRIBUTED
NATURE OF NUCLEUS (NOT POINT CHARGE)
AND θ_{\min} ARISES FROM SCREENING OR ELECTRONS
OR UNCERTAINTY PRINCIPLE

$$\ln \frac{\theta_{\max}}{\theta_{\min}} \approx \ln(204 Z_g^{1/3})$$

$$\Delta \langle x'^2 \rangle = \frac{1}{2} \overline{\text{H}^2} \quad [\text{since } \Delta \langle x'^2 \rangle + \Delta \langle y'^2 \rangle = \Delta \text{H}^2]$$

$$\epsilon = 4 \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

FOR A MATCHED BEAM

$$k_p^2 \langle x^2 \rangle = \langle x'^2 \rangle \quad \text{where } k_p = \text{depressed wavenumber}$$

$$\Rightarrow \epsilon = \frac{4 \langle x'^2 \rangle}{k_p}$$

$$\Delta \epsilon = \frac{4 \Delta \langle x'^2 \rangle}{k_p} = \frac{2 \overline{\text{H}^2}}{k_p}$$

$$\Rightarrow \frac{d\epsilon}{ds} = \frac{32 \pi}{k_p} n_g \left(\frac{Z Z_g e^2}{4 \pi \epsilon_0 m c^2 k_p^2} \right)^2 \ln(204 Z_g^{-1/3})$$

IN TERMS OF NORMALIZED EMISSIONANCE:

$$\frac{dE_N}{ds} = \frac{32 \pi}{k_p} n_g \left(\frac{Z Z_g e^2}{4 \pi \epsilon_0 m c^2} \right)^2 \frac{1}{\delta p^3} \ln(204 Z_g^{-1/3})$$

Example: $n_g = 10^{-7} \text{ torr} = 3.5 \cdot 10^9 \text{ cm}^{-3} = 3.5 \cdot 10^{15} \text{ m}^{-3}$

$$k_{p0} = 2.5 \text{ m}^{-1} \quad k_p = 0.25 \text{ m}^{-1}$$

$$Z_g = 7, Z = 19, A = 39, \beta = 0.01, \epsilon_N = 1 \cdot 10^{-6} \text{ m} \cdot \text{rad}$$

$$\Rightarrow \frac{dE_N}{ds} = 3.7 \cdot 10^{-11} \text{ m}^{-1} \Rightarrow \text{Need 27 km to equal original emittance!}$$

(But more important for rings & low mass!)

Pressure "Pumps" AND

BEAM LOSS FROM CHARGE CHANGING COLLISIONS

Ref. RECENT WORKSHOP ON BEAM INDUCED PRESSURE PUMPS IN LINGS, BNL, Dec. 2003,

σ_s = STRIKING CROSS SECTION

σ_{ce} = CHARGE EXCHANGE CROSS SECTION

σ_i = IONIZATION CROSS SECTION

v_{cm} = mean ion velocity
in ion beam frame

(1) BEAM LOSS

$$\frac{dN_b}{dt} = -\sigma_s v_i N_b \bar{n} - \sigma_{ce} v_{cm} N_b^2 - \frac{dN_b}{dt} \Big|_{Halo}$$

(2) GAS EVOLUTION

\bar{n} = average gas density

STRIKING

$$\frac{d\bar{n}}{dt} = \gamma_0 \sigma_i v_i N_b \bar{n} \left(\frac{V_{beam}}{V_{pipe}} \right) + \gamma_{H^+} \sigma_s v_i N_b \bar{n} \left(\frac{V_{beam}}{V_{pipe}} \right) + \gamma_{H^-} \sigma_a v_{cm} N_b^2 \left(\frac{V_{beam}}{V_{pipe}} \right) + q - (S/A_p) \bar{n}$$

CHARGE EXCHANGE

OUTGASSING

PUMPING

$$S = \text{linear pumping rate } [\text{m}^2 \text{s}^{-1}/\text{m}]$$

$$A_p = \pi r_p^2 = \text{area of pipe}$$

$$q = \text{outgassing rate} = \frac{2\pi r_p Q}{\pi r_p^2} = \frac{2Q}{r_p}; \quad Q = \frac{\#}{\text{m}^2 \text{s}}$$

N_b = GAS MOLECULES DESORBED FOR INCIDENT RESIDUAL GAS ION

N_{ce} = GAS MOLECULES DESORBED FOR INCIDENT IONIZATION STRIKING WALL

$$(V_{beam}/V_{pipe}) V_{ref} \delta t_{beam} \text{ for a ref rated linac}$$

If we take $n_b \approx \text{constant}$

then we may express gas evolution equation as:

$$\frac{d\bar{n}}{dt} = \frac{\bar{n}}{\tau} + q_{\text{eff}}$$

with solution:

$$\bar{n} = (\bar{n}_0 + \tau q_{\text{eff}}) \exp[t/\tau] - \tau q_{\text{eff}}$$

HERE $\tau = \frac{1}{(\eta_g O_i + \eta_{H_2} O_S) \left(\frac{V_{\text{beam}}}{V_{\text{pipe}}} \right) n_b V_i - S/A_p}$

$$q_{\text{eff}} = q + \eta_{H_2} O_{cr} V_{cm} n_b^2 \left(\frac{V_{\text{beam}}}{V_{\text{pipe}}} \right)$$

EQUILIBRIUM REACHED IF $\tau < 0$ (i.e. pumping exceeds desorption).

$$\Rightarrow \bar{n} = -\tau q_{\text{eff}} = \frac{q + \eta_{H_2} O_{cr} V_{cm} n_b^2 (V_{\text{beam}}/V_{\text{pipe}})}{S/A_p - (\eta_g O_i + \eta_{H_2} O_S) \left(\frac{V_{\text{beam}}}{V_{\text{pipe}}} \right) n_b V_i}$$

INSTABILITY IF

$$n_b V_i > \frac{S \left(\frac{V_{\text{beam}}}{V_{\text{pipe}}} \right)}{\eta_g O_i + \eta_{H_2} O_S}$$

Instability first observed on the ISR proton storage ring, limiting current in rings, in 1970's.

If $I_{beam} = I_{pipe}$

INSTABILITY CRITERION MAY BE WRITTEN

$$I > \frac{zeS}{\gamma_B \Omega_t + \gamma_{He} \Omega_s}$$

EXAMPLE: If $S = 100 \text{ ls}^{-1} \text{ m}^{-1} = 0.1 \text{ m}^3 \text{ s}^{-1} \text{ m}^{-1}$

ISR

$$\gamma_B \approx 4$$

$$\Omega_t = 10^{-22} \text{ m}^2 = 10^{-16} \text{ cm}^2 ; \quad \Omega_s = 0$$

$$z = 1 \quad (\text{protons})$$

$$\Rightarrow I \lesssim 40 \text{ Amperes}$$

(PRESSURE RUNAWAYS were OBSERVED on the ISR AT 14-18A,
(BENVENUTI et al, IEEE Trans. on Nucl. Sci. NS-24, 1973, 1977)

SEE "BEAM INDUCED PRESSURES RISE IN RINGS"

13th ICFA BEAM DYNAMICS MINI WORKSHOP, BNL, Dec 9-12, 2003.
WEBSITE: <http://www.c-ad.bnl.gov/icfa>

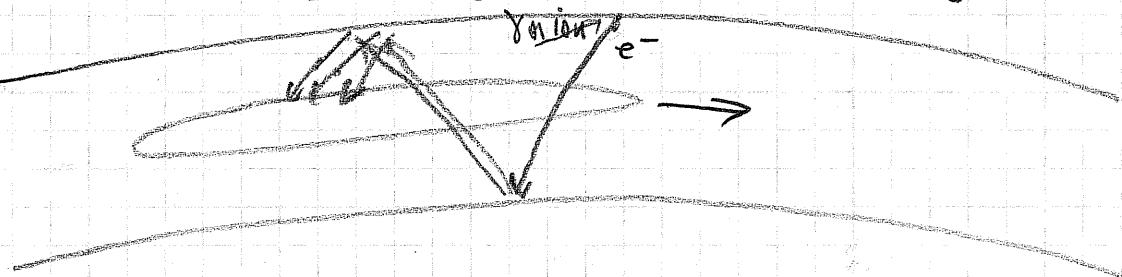
"ELECTRON CLOUD EFFECTS"

REFERENCE: CERN e-CLOUD WORKSHOP

<http://wwwslap.cern.ch/collective/ecloudphi2/> → proceedings.html

BASIC IDEA

IN ION storage rings or collider rings:



ELECTRONS ARE ATTRACTED TO POSITIVE POTENTIAL
OF BEAM & ACCUMULATE

SOME SYMPTOMS:

1. Beam loss & pressure rise
2. High frequency control oscillations

SOME ACCELERATORS WHICH SHOW EVIDENCE OF e- EFFECTS

1. LANL PSR
2. CERN PS & SPS
3. BNL RHIC

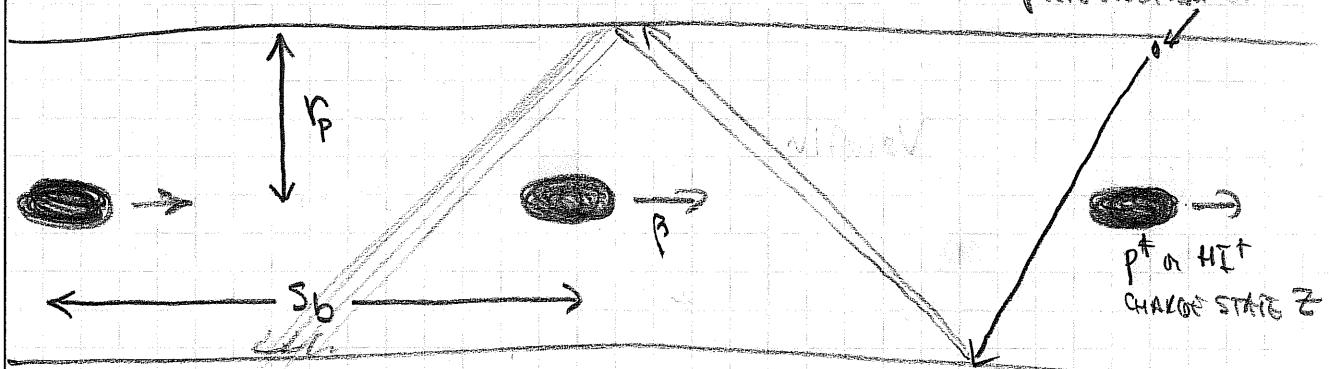
COLLIDER RINGS / UNDER CONSTRUCTION:

1. SNS ACCUMULATOR RING
2. LHC

c.f. "Electron-cloud effects in"

HIGH INTENSITY PROTON ACCELERATORS
J. Wei & R. Macak, CERNBEAM INDUCED MULTIPACTING

a) MULTI-BUNCH



$$\Delta p_x \approx \frac{2Z N_b e^2}{4\pi\epsilon_0 V r_p}$$

N_b = Number of ions of charge Z
in bunch

$$\Delta E_e = m_e c^2 \left[\sqrt{\frac{\Delta p_x^2}{m_e c^2}} + 1 - 1 \right] = m_e c^2 \left[\sqrt{\left(\frac{2 Z e N_b}{\beta r_p} \right)^2 + 1} - 1 \right]$$

$$\approx \frac{2 r_e m_e c^2 Z^2 N_b^2}{\beta^2 r_p^2} \quad \text{for } \Delta E_e \ll m_e c^2$$

$\left(\text{or } \frac{2 r_e Z N_b}{\beta r_p} \ll 1 \right)$

DEFINE A MULTIPACTING PARAMETER

 f_m

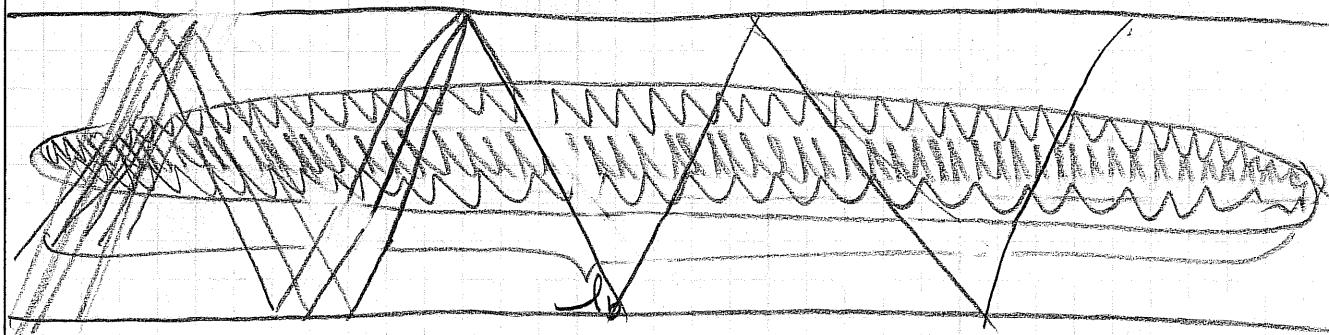
$$f_m = \frac{\text{TIME FOR ELECTRON TO CROSS RAY}}{\text{TIME BETWEEN BUNCHES}} = \frac{2 r_p \beta}{S_b \beta c}$$

$$\approx \frac{\beta^2 r_p^2}{Z N_b r_e S_b}$$

RESONANCE CONDITION:

$$f_m = 1$$

b) SINGLE-BUNCH Beam-Induced Multipacting



$$\int_s = \frac{r_p b}{l_b \beta_c} = \frac{\text{time for electrons to cross pipe}}{\text{passage time for half of the bunch}}$$

Recall:

$$\psi = \begin{cases} \frac{\lambda}{2\pi\epsilon_0} \left[\frac{1}{2} \left(1 - \frac{r^2}{r_b^2} \right) + \ln \frac{r}{r_b} \right] & 0 < r < r_b \\ \frac{\lambda}{2\pi\epsilon_0} \left[\ln \frac{r}{r_b} \right] & r_b < r < r_p \end{cases}$$

$$\frac{1}{2} m_e v_e^2 + q\phi \approx \text{const} \approx 0$$

(AVENAGE
e- VELOCITY)

$$\beta_c \approx \frac{1}{2} \sqrt{\frac{2q\phi}{m_e c^2}} \approx \sqrt{\frac{N_e Z e^2}{l_b 4\pi\epsilon_0 m_e c^2}} \approx \sqrt{\frac{Z N_e N_o}{l_b}}$$

$$\Rightarrow \int_s = \frac{B r_r}{V r_{lb} N_o Z}$$

THE ENERGY GAIN OF THE ELECTRON, RELIES ON THE DENSITY CHANGING OVER THE COURSE OF THE BUNCH.

$$\Delta E_e \approx \frac{m_e c^2}{2} \left[\frac{Z N_e N_o(l)}{l_b} \right] - \frac{m_e c^2}{2} \left[\frac{Z N_e N_o(s+\delta s)}{l_b} \right]$$

$$\approx \frac{m_e c^2}{2} \left(\frac{\partial N_o}{\partial z} \Delta z \right) \left(\frac{Z r_r}{l_b} \right)$$

$$\Delta E_e \sim \frac{mc^2}{2} \left(\frac{\partial N_b}{\partial z} \Delta z \right) \left(\frac{Z r_e}{l_b} \right)$$

$$\Delta z = \frac{V_p \rho c}{\chi_e} = \beta r_p \sqrt{\frac{l_b}{Z r_e N_b}};$$

$$\frac{\partial N_b}{\partial z} \sim \frac{N_b}{l_b}$$

$$\text{So } \Delta E_e \sim mc^2 \left(\frac{Z N_b V_p}{l_b^3} \right)^{1/2}$$

$\zeta \ll 1 \Rightarrow$ Electron build up
possible within bunch

WHAT IS STEADY STATE ELECTRON DENSITY?

Electrons can build up until E_n at pipe ≈ 0 .

$$\Rightarrow \lambda_e = \lambda_J$$

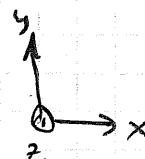
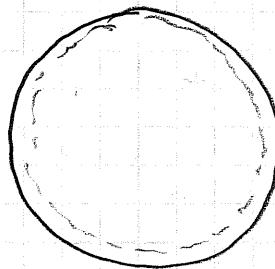
$$\pi r_p^2 n_e = \pi r_b^2 Z n_i$$

$$n_e = \left(\frac{r_b}{r_p} \right)^2 Z n_i$$

ELECTRON - ION INSTABILITY

(SEE ALSO R.C.DAVIDSON
& H.QIN, Physics of Intense
Charged Particle Beams (in
High Energy Accelerators), p.503
FOR KINETIC TREATMENT).

CONSIDER A UNIFORM DISTRIBUTION OF ELECTRONS (AT REST)
WHICH HAS THE SAME RADIUS (OR SLIGHTLY SMALLER, RADIUS)
AS A UNIFORM DENSITY ION BEAM, THAT IS MOVING AT VELOCITY
 v_z (OUT OF THE PLANE OF THE PAPER).



THE EQUATION OF MOTION FOR THE CENTROID OF THE
ELECTRONS IS OBTAINED FROM

$$\frac{d^2 x_e}{dt^2} = e E_{sx} \quad \text{where } E_{sx} = \text{space charge field}$$

of the ion beam

$$(5) \quad \frac{d^2 x_e}{dt^2} = - \frac{\omega_{pi}^2}{2} \left(\frac{m_i e}{q^2 m_e} \right) (x_e - x_i)$$

$$\text{here } \omega_{pi}^2 = \frac{q^2 n_i}{\epsilon_0 m_i}$$

(THE CENTER OF OSCILLATION FOR THE ELECTRONS IS THE
CENTER OF THE ION BEAM).

THE EQUATION OF MOTION FOR THE CENTROID OF THE IONS IS GIVEN BY

$$\frac{d^2x_i}{dt^2} = -\omega_{pi}^2 x_i - \left[\frac{m_e N_e}{m_i N_i} \right] \left(\frac{\omega_{pi}^2}{2} \frac{m_i e}{q m_e} \right) (x_i - x_e)$$

↑
THE TOTAL MOMENTUM
KICK TO EACH SPECIES
MUST BE EQUAL & OPPOSITE

∴ $\frac{d^2x_i}{dt^2} = -\omega_{pi}^2 x_i - f \frac{\omega_{pi}^2}{2} (x_i - x_e)$

HERE $f = \frac{e N_e}{q N_i}$ = fractional neutralization

Now $\frac{d}{dt} = \text{total derivative} = \frac{\partial}{\partial t} - v_z \frac{\partial}{\partial z}$

⇒ THE ION & ELECTRON EQUATIONS MAY BE WRITTEN

$$\left(\frac{\partial^2}{\partial t^2} - 2v_z \frac{\partial}{\partial z} + v_z^2 \frac{\partial^2}{\partial z^2} \right) x_i = -\omega_{pi}^2 x_i - f \frac{\omega_{pi}^2}{2} (x_i - x_e)$$

$$\frac{\partial^2}{\partial t^2} x_e = -\frac{\omega_{pi}^2}{2} \left(\frac{m_i e}{q m_e} \right) (x_e - x_i)$$

Now let $x_e = x_0 \exp[i(\omega t - kz)]$; $x_i = x_i \exp[i(\omega t - kz)]$

$$\Rightarrow (-\omega^2 + 2\omega k V_z - k^2 V_z^2) x_i = -\omega_{pi}^2 x_i - f \frac{\omega_{pi}^2}{2} (x_i - x_e)$$

$$-\omega^2 x_e = -\frac{\omega_{pi}^2}{2} \left(\frac{m_i e}{m_e q} \right) (x_e - x_i)$$

$$\Rightarrow \left[(\omega - kV_z)^2 - \omega_{pi}^2 + f \frac{\omega_{pi}^2}{2} \right] x_i = \frac{f \omega_{pi}^2}{2} x_e$$

$$\left[\omega^2 - \frac{\omega_{pi}^2}{2} \left(\frac{m_i e}{m_e q} \right) \right] x_e = \frac{\omega_{pi}^2}{2} \left(\frac{m_i e}{m_e q} \right) x_i$$

Multiplying the above equations and dividing by $x_i x_e$, yields the dispersion relation:

$$\underbrace{\left[(\omega - kV_z)^2 - \omega_{pi}^2 + f \frac{\omega_{pi}^2}{2} \right]}_{\text{ION BETATRON FREQUENCY (REFINED BY SPACE CHARGE OF ELECTRON)}} \underbrace{\left[\omega^2 - \frac{\omega_{pi}^2}{2} \left(\frac{m_i e}{m_e q} \right) \right]}_{\text{ELECTRON OSCILLATING POTENTIAL WELL OF ION}} = \underbrace{f \frac{\omega_{pi}^2}{4} \left(\frac{m_i e}{m_e q} \right)}_{\text{COUPLING}}$$

ION BETATRON FREQUENCY (REFINED BY SPACE CHARGE OF ELECTRON)

ELECTRON
OSCILLATING
POTENTIAL
WELL OF ION

COUPLING

If a beam with high spatial frequency satisfy betatron oscillations in the coupling frame, $\omega - kV_z \approx \sqrt{\omega_{pi}^2 + f \frac{\omega_{pi}^2}{2}}$

will resonate with electrons oscillating in the ion well if

$$\omega \approx \frac{\omega_{pi}}{\sqrt{2}} \sqrt{\frac{m_i e}{m_e q}}$$

Giving rise to instabilities!

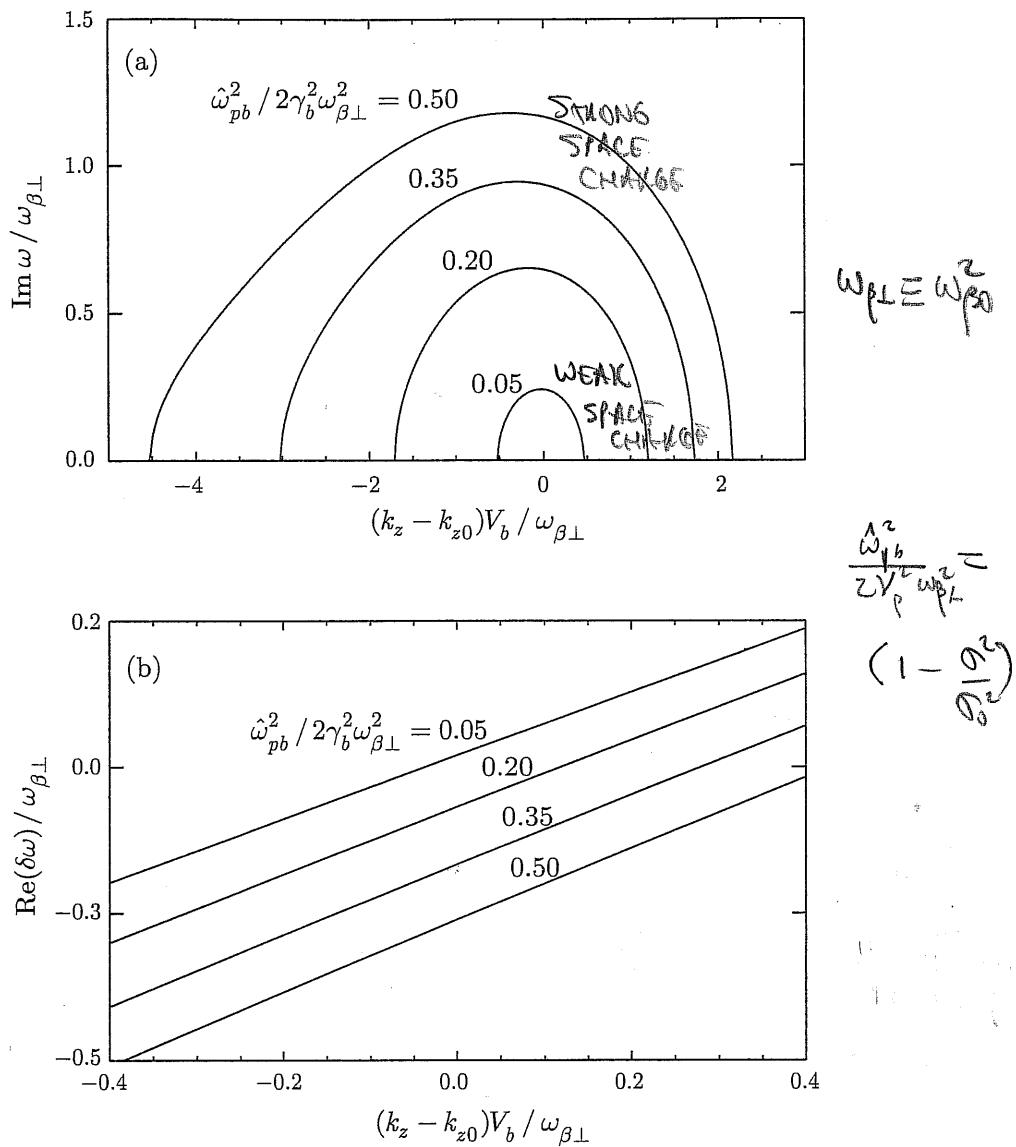


Figure 10.11. Plots of (a) normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and (b) normalized real frequency ($Re(\delta\omega) - \omega_e$)/ $\omega_{\beta b}$ versus shifted axial wavenumber $(k_z - k_{z0})V_b/\omega_{\beta\perp}$ obtained from the dispersion relation (10.103) for the unstable branch with positive real frequency. System parameters correspond to $v_{T\parallel b} = 0 = v_{T\parallel e}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_b c^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized beam intensity $\hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2$ ranging from 0.05 to 0.5.

$$k_{z0} V_z = \omega + \sqrt{\omega_{pe}^2 + f m_i^2 c^2 / 2} ; \quad \omega = \frac{m_i e}{2} \sqrt{\frac{m_i e}{m_e q}}$$

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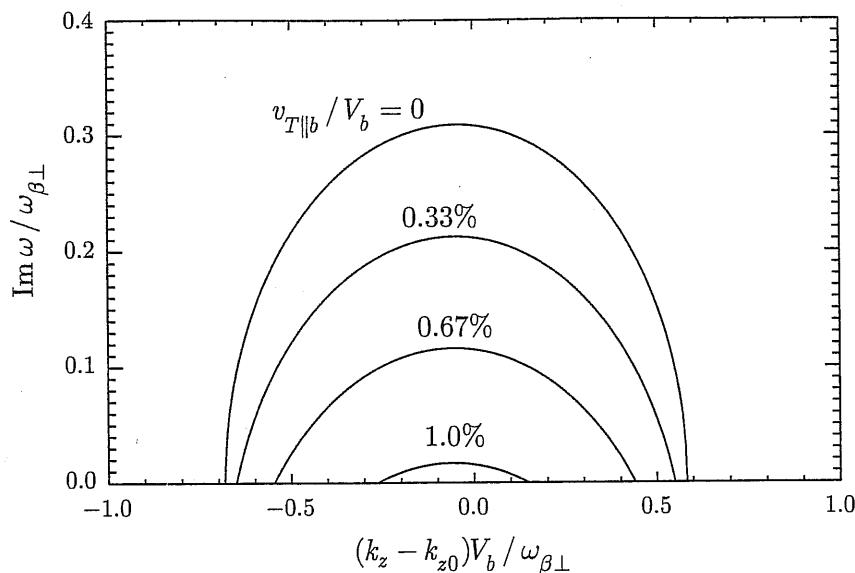


Figure 10.12. Plot of normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and normalized real frequency ($(k_z - k_{z0})V_b/\omega_{\beta\perp}$) versus positive real frequency. System parameters correspond to $\hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2 = 0.07$, $v_{T\parallel e} = v_{T\parallel b}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_b c^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized ion thermal spread $v_{T\parallel b}/V_b$ ranging from 0 to 0.01.

velocity V_b [Eq. (10.105)], it is expected that Landau damping by parallel ion kinetic effects can have a strong stabilizing influence at modest values of $v_{T\parallel b}/V_b$. That this is indeed the case is evident from Fig. 10.12, which shows a substantial reduction in maximum growth rate and eradication of the instability over the instability bandwidth as $v_{T\parallel b}/V_b$ is increased from 0 to 0.01.

We now make use of the nonlinear particle perturbative simulation method [60, 61] described in Sec. 8.5 to illustrate several important properties of the two-stream instability in intense beam systems (Sec. 10.4.2).

PREVENTIVE MEASURES (from J. Weid L. Marek, CERN electron cloud workshop 2003)

- SUPPRESS ELECTRON GENERATION
- SURFACE TREATMENT OF THE VACUUM FLANGE
- KICKED MAGNETS IN GAPS
- VACUUM VOLTS SCREENED TO COVER E-FID
- CLEANING ELECTRODES
- HIGH VACUUM
- SOLENOIDS - TO REDUCE MULTIVATING

Summary of Emission, Gas, Pressure, & Scattering Effect

1. COULOMB COLLISIONS WITHIN BEAM CAN TRANSFER ENERGY FROM L TO H AND PROVIDE LOWER LIMIT ON $T_{H,i}$, HIGHER THAN FROM ACCUMULATIVE COOLING.
2. COULOMB INTERACTIONS WITH RESIDUAL GAS NUCLEI PROVIDE A SOURCE OF EMITTANCE GROWTH (BUT NOT IMPORTANT FOR HIGHER MASS AND LONGER RETENTION TIMES).
3. PRESSURE INSTABILITY FROM DESORPTION OF RESIDUAL GAS BY STRIKED BEAM IONS Hitting wall OR BEAM-IONIZED RESIDUAL GAS ATOMS, FORCED TO WALL BY E-FIELD OF BEAM. LIMITS CURRENT IN KINGS OR HIGH VOLTAGE LINAC.
4. ELECTRONS CAN CASCADE AND REACH A "QUASI" EQUILIBRIUM VOLATILIZATION OF SIMILAR LINE CHARGE TO THE ION BEAM. ELECTRON-ION TWO STREAM INSTABILITY IS UNSTABLE, AND CAN LEAD TO TRANSVERSE INSTABILITY, SIMILAR TO WHAT IS OBSERVED IN SOME KTON KINGS.